

# Quark Transverse Momentum Distributions inside a nucleon : a Light-Front Hamiltonian Dynamics study <sup>\*</sup>

EMANUELE PACE

Università di Roma “Tor Vergata” and INFN, Roma 2, Italy

GIOVANNI SALME<sup>†</sup>

INFN Sezione di Roma, Italy

SERGIO SCOPETTA

Università di Perugia and INFN, Sezione di Perugia, Italy

ALESSIO DEL DOTTO

Università di Roma Tre and INFN, Roma 3, Italy

Through an impulse approximation analysis of single spin Sivers and Collins asymmetries in the Bjorken limit, the possibility to extract the quark transverse-momentum distributions in the neutron from semi-inclusive deep inelastic electron scattering off polarized  ${}^3\text{He}$  is illustrated. The analysis is generalized to finite momentum transfers in a light-front Poincaré covariant framework, defining the light-front spin-dependent spectral function of a  $J=1/2$  system. The definition of the light-front spin-dependent spectral function for constituent quarks in the nucleon allows us to show that, within the light-front dynamics, only three of the six leading twist T-even transverse-momentum distributions are independent.

PACS numbers: 12.39.Ki,13.40.-f,13.60.Hb,21.45.Bc,23.30.Fj

## 1. Introduction

The interest in the transverse momentum-dependent parton distributions (TMDs) inside a nucleon is a consequence of the “spin crisis”: most of the proton spin is carried by the quark orbital angular momentum,  $L_q$ , and by the gluons. Information on the TMDs [1] and then on  $L_q$  can be accessed

---

<sup>\*</sup> Presented at Light Cone 2012

through non forward processes, as semi-inclusive deep inelastic electron scattering (SIDIS). In particular single spin asymmetries SSAs in the scattering of an unpolarized electron beam on a transversely polarized target allow one to distinguish the Sivers and the Collins asymmetries, which can be expressed in terms of different TMDs and fragmentation functions (ff) [1, 2]. Actually a large Sivers asymmetry was measured in the process  $\vec{p}(e, e'\pi)x$  [3], while only a small Sivers asymmetry was measured in  $\vec{D}(e, e'\pi)x$  [4]. Then one can infer a strong flavour dependence of the asymmetries, confirmed by recent data [5]. This puzzle has attracted a great interest in obtaining new information on the neutron TMDs.

In Ref. [6] the possibility was proposed to extract information on the neutron TMDs from experimental measurements of the single spin asymmetries on  $^3\text{He}$ . In Ref. [2] an impulse approximation (IA) approach to SIDIS off  $^3\text{He}$  in the Bjorken limit was presented and applied to demonstrate how neutron Sivers and Collins SSAs can be extracted from the  $^3\text{He}$  and proton ones.

In this contribution we develop a light-front approach to SIDIS on  $^3\text{He}$ , already considered in [7], to take care of relativistic effects at finite values of the momentum transfer through a light-front spectral function for  $^3\text{He}$ .

Eventually, through the definition of the light-front spin-dependent spectral function for a nucleon, considered as a system of three quarks, we show that within the light-front dynamics only three of the six leading twist time-reversal-even transverse-momentum distributions are independent.

## 2. Neutron single-spin asymmetries and a polarized $^3\text{He}$ target

As is well known, a polarized  $^3\text{He}$  is an ideal target to study the neutron, since at a 90% level a polarized  $^3\text{He}$  is equivalent to a polarized neutron. In order to take care of the motion of the bound nucleons in  $^3\text{He}$ , dynamical nuclear effects in inclusive deep inelastic electron scattering processes,  $^3\vec{H}e(e, e')X$ , (DIS) were evaluated with a realistic spin-dependent spectral function (SDSF) for  $^3\vec{H}e$  [8]. It was found that the formula

$$A_n \simeq \frac{1}{p_n f_n} \left( A_3^{exp} - 2p_p f_p A_p^{exp} \right), \quad (f_p, f_n \text{ dilution factors}) \quad (1)$$

can be safely adopted to extract the neutron information from  $^3\text{He}$  and proton data. Actually this formula is widely used by experimental collaborations to this end. All the nuclear effects are hidden in the proton and neutron "effective polarizations"  $p_p$  and  $p_n$ . In [2] the values  $p_p = -0.023$ ,  $p_n = 0.878$  were obtained for the AV18 interaction [9] using the overlaps calculated in [10]. Only negligible effects can be found including three-body interactions.

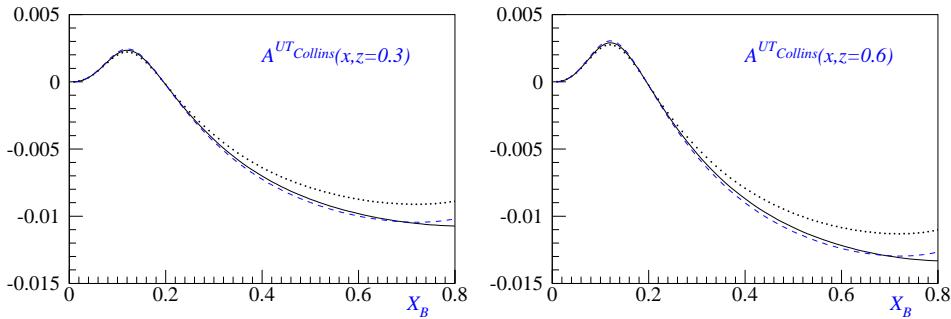


Fig. 1. Collins asymmetry. Full line: neutron asymmetry of the adopted model,  $A_n^{model}$ ; dotted line: neutron asymmetry extracted from the calculated  ${}^3He$  asymmetry neglecting the proton polarization contribution:  $\bar{A}_n \simeq \frac{1}{f_n} A_3^{calc}$ ; dashed line: neutron asymmetry extracted from the calculated  ${}^3He$  asymmetry taking into account nuclear structure through Eq. (1) (after [2]).

To investigate if an analogous formula can be used to extract the SSAs, in [2] the process  ${}^3He(e, e'\pi)X$  was evaluated in the Bjorken limit and in impulse approximation, i.e. the final state interaction (FSI) was considered only between the two-nucleon system which recoils, while no interaction was considered between the measured fast, ultrarelativistic  $\pi$  and the remnant. In IA, SSAs for  ${}^3He$  involve convolutions of SDSF for  ${}^3He$  with TMDs and fragmentation functions. The nuclear effects on ff are new with respect to the DIS case. Ingredients of the calculations were: i) a realistic SDSF for  ${}^3He$  [11, 10], obtained using the AV18 interaction and the wave functions evaluated by the Pisa group [12]; ii) parametrizations of data for TMDs and ff, whenever available; iii) models for the unknown TMDs and ff. As shown in Fig. 1, in the Bjorken limit the extraction procedure through the formula successful in DIS works nicely for the Collins SSA as well, replacing in Eq. (1)  $A_3^{exp}$  with the calculated  ${}^3He$  asymmetry,  $A_3^{calc}$ , and  $A_p^{exp}$  with the proton asymmetry corresponding to the adopted model,  $A_p^{model}$ . The same was shown to occur for the Sivers SSA [2].

In [2] the calculation was performed in the Bjorken limit. To study relativistic effects in the actual experimental kinematics, in Ref. [7] we adopted the light-front (LF) form of Relativistic Hamiltonian Dynamics (RHD) introduced by Dirac. Indeed the RHD of an interacting system with a *fixed number* of on-mass-shell constituents, *plus* the Bakamjian-Thomas construction of the Poincaré generators allow one to generate a description of SIDIS off  ${}^3He$  which is fully Poincaré covariant.

Within the LF Hamiltonian dynamics one has 7 kinematical generators, a subgroup structure of the LF boosts, a separation of the intrinsic and the center of mass motion and a meaningful Fock expansion.

In IA the LF hadronic tensor for the  ${}^3\text{He}$  nucleus is:

$$\begin{aligned} \mathcal{W}^{\mu\nu}(Q^2, x_B, z, \tau, \hat{\mathbf{h}}, S_{He}) \propto & \sum_{\sigma, \sigma'} \sum_{\tau} \sum_{\epsilon_S^{\min}}^{\epsilon_S^{\max}} d\epsilon_S \int_{M_N^2}^{(M_X - M_S)^2} dM_f^2 \quad (2) \\ & \times \int_{\xi_{low}}^{\xi_{up}} \frac{d\xi}{\xi^2(1-\xi)(2\pi)^3} \int_{P_{\perp}^m}^{P_{\perp}^M} \frac{dP_{\perp}}{\sin\theta} (P^+ + q^+ - h^+) w_{\sigma\sigma'}^{\mu\nu}(\tau, \tilde{\mathbf{q}}, \tilde{\mathbf{h}}, \tilde{\mathbf{P}}) \\ & \times \mathcal{P}_{\sigma'\sigma}^{\tau}(\mathbf{k}, \epsilon_S, S_{He}) \end{aligned}$$

where  $\tilde{\mathbf{v}} = \{v^+ = v^0 + v^3, \mathbf{v}_{\perp}\}$ ,  $w_{\sigma\sigma'}^{\mu\nu}(\tau, \tilde{\mathbf{q}}, \tilde{\mathbf{h}}, \tilde{\mathbf{P}})$  is the nucleon hadronic tensor and  $\mathcal{P}_{\sigma'\sigma}^{\tau}(\mathbf{k}, \epsilon_S, S_{He})$  the LF spectral function for  ${}^3\text{He}$  given in terms of the unitary Melosh Rotations,  $D^{\frac{1}{2}}[\mathcal{R}_M(\mathbf{k})]$ , and of the instant-form spectral function  $\mathcal{S}_{\sigma_1'\sigma_1}^{\tau}(\mathbf{k}, \epsilon_S, S_{He})$  [13]:

$$\mathcal{P}_{\sigma'\sigma}^{\tau}(\mathbf{k}, \epsilon_S, S_{He}) \propto \sum_{\sigma_1\sigma_1'} D^{\frac{1}{2}}[\mathcal{R}_M^{\dagger}(\mathbf{k})]_{\sigma'\sigma_1'} \mathcal{S}_{\sigma_1'\sigma_1}^{\tau}(\mathbf{k}, \epsilon_S, S_{He}) D^{\frac{1}{2}}[\mathcal{R}_M(\mathbf{k})]_{\sigma_1\sigma} \quad (3)$$

Notice that  $\mathcal{S}_{\sigma_1'\sigma_1}^{\tau}$  is given in terms of three independent functions,  $B_{0,He}^{\tau}, B_{1,He}^{\tau}, B_{2,He}^{\tau}$  [11]

$$\mathcal{S}_{\sigma_1'\sigma_1}^{\tau}(\tilde{\mathbf{k}}, \epsilon_S, S_{He}) = \left[ B_{0,He}^{\tau}(|\mathbf{k}|, E) + \boldsymbol{\sigma} \cdot \mathbf{f}_{S_{He}}^{\tau}(\mathbf{k}, E) \right]_{\sigma_1'\sigma_1} \quad (4)$$

with  $\mathbf{f}_{S_{He}}^{\tau}(\mathbf{k}, E) = \mathbf{S}_{He} B_{1,He}^{\tau}(|\mathbf{k}|, E) + \hat{k} (\hat{k} \cdot \mathbf{S}_A) B_{2,He}^{\tau}(|\mathbf{k}|, E)$  and  $\mathbf{S}_{He}$  the  ${}^3\text{He}$  polarization vector.

We are now evaluating the SSAs using the LF hadronic tensor at finite values of  $Q^2$  (Eq. (2)). The preliminary results are quite encouraging. Indeed, as shown in the Table, LF longitudinal and transverse polarizations only weakly differ and the differences with respect to the non-relativistic results are small. Furthermore, we find that in the Bjorken limit the extraction procedure works well within the LF approach as it does in the non-relativistic case. The effect of the finite integration limits in the actual JLAB kinematics [14], instead of the ones in the Bjorken limit, is small and will be even smaller in the JLAB planned experiments at 12 GeV [6].

Concerning the FSI, we plan to include the FSI between the jet produced from the hadronizing quark and the two-nucleon recoiling system through a Glauber approach [15].

	proton NR	proton LF	neutron NR	neutron LF
$\int dE d\vec{p} \frac{1}{2} \text{Tr}(\mathcal{P} \sigma_z)_{\tilde{S}_A = \hat{z}}$	-0.02263	-0.02231	0.87805	0.87248
$\int dE d\vec{p} \frac{1}{2} \text{Tr}(\mathcal{P} \sigma_y)_{\tilde{S}_A = \hat{y}}$	-0.02263	-0.02268	0.87805	0.87494

### 3. The $J = 1/2$ LF spectral function and the nucleon LF TMDs

The TMDs for a  $J = 1/2$  system are introduced through the q-q correlator (with  $\psi_\alpha$  the  $\alpha$  Dirac component of the quark field) [1]

$$\Phi_{\alpha\beta}(k, P, S) = \int d^4z e^{ik\cdot z} \langle PS| \bar{\psi}_\beta(0) \psi_\alpha(z) |PS\rangle \quad (5)$$

$$\Phi(k, P, S) = \frac{1}{2} \left\{ A_1 \not{P} + A_2 S_L \gamma_5 \not{P} + A_3 \not{P} \gamma_5 S_\perp + \frac{1}{M} \tilde{A}_1 \vec{k}_\perp \cdot \vec{S}_\perp \gamma_5 \not{P} + \right. \\ \left. + \tilde{A}_2 \frac{S_L}{M} \not{P} \gamma_5 \not{k}_\perp + \frac{1}{M^2} \tilde{A}_3 \vec{k}_\perp \cdot \vec{S}_\perp \not{P} \gamma_5 \not{k}_\perp \right\}, \quad (6)$$

so that the six twist-2 T-even TMDs,  $A_i, \tilde{A}_i$  ( $i = 1, 3$ ), can be obtained by proper traces of  $\Phi(k, P, S)$ . Let us consider the contribution to the correlation function from on-mass-shell fermions

$$\Phi_p(k, P, S) = \frac{(\not{k}_{on} + m)}{2m} \Phi(k, P, S) \frac{(\not{k}_{on} + m)}{2m} = \quad (7)$$

$$= \sum_{\sigma} \sum_{\sigma'} u_{LF}(\tilde{k}, \sigma') \bar{u}_{LF}(\tilde{k}, \sigma') \Phi(k, P, S) u_{LF}(\tilde{k}, \sigma) \bar{u}_{LF}(\tilde{k}, \sigma)$$

and let us identify  $\bar{u}_{LF}(\tilde{k}, \sigma') \Phi(k, P, S) u_{LF}(\tilde{k}, \sigma)$  with the LF nucleon spectral function,  $\mathcal{P}_{\sigma'\sigma}(\tilde{\mathbf{k}}, \epsilon_S, S)$ . The off-mass-shell minus component  $k^-$  of the struck quark is related to the spectator diquark energy  $\epsilon_S$  [13]. The traces of  $[\gamma^+ \Phi_p(k, P, S)]$ ,  $[\gamma^+ \gamma_5 \Phi_p(k, P, S)]$ , and  $[\not{k}_\perp \gamma^+ \gamma_5 \Phi_p(k, P, S)]$  can be obtained in terms of the TMD's,  $A_i, \tilde{A}_i$  ( $i = 1, 3$ ), through Eq. (5). However these same traces can be also expressed through the LF nucleon spectral function, since

$$\frac{1}{2P^+} \text{Tr} [\gamma^+ \Phi_p(k, P, S)] = \frac{k^+}{2mP^+} \text{Tr} [\mathcal{P}(\tilde{\mathbf{k}}, \epsilon_S, S)] \quad (8)$$

$$\frac{1}{2P^+} \text{Tr} [\gamma^+ \gamma_5 \Phi_p(k, P, S)] = \frac{k^+}{2mP^+} \text{Tr} [\sigma_z \mathcal{P}(\tilde{\mathbf{k}}, \epsilon_S, S)] \quad (9)$$

$$\frac{1}{2P^+} \text{Tr} [\not{k}_\perp \gamma^+ \gamma_5 \Phi_p(k, P, S)] = \frac{k^+}{2mP^+} \text{Tr} [\mathbf{k}_\perp \cdot \boldsymbol{\sigma} \mathcal{P}(\tilde{\mathbf{k}}, \epsilon_S, S)] \quad (10)$$

In turn the traces  $\frac{1}{2} \text{Tr}(\mathcal{P}I)$ ,  $\frac{1}{2} \text{Tr}(\mathcal{P}\sigma_z)$ ,  $\frac{1}{2} \text{Tr}(\mathcal{P}\sigma_i)$  ( $i = x, y$ ) can be expressed in terms of known kinematical factors and three scalar functions,  $B_{0,N}, B_{1,N}, B_{2,N}$ , since the nucleon spectral function is given in terms of these functions, in analogy with the  ${}^3\text{He}$  case.

Then in the LF approach with a *fixed number* of particles the six leading twist TMDs,  $A_i, \tilde{A}_i$  ( $i = 1, 3$ ), can be expressed in terms of the previous three independent scalar functions.

#### 4. Conclusion

An realistic study in impulse approximation of the process  ${}^3\vec{H}e(e, e'\pi)X$  in the Bjorken limit has been described. The effect of nuclear structure in the extraction of the neutron asymmetries was found to be under control. The extraction procedure of the neutron information from  ${}^3\vec{H}e(e, e'\pi)X$  experiments at finite  $Q^2$  is now being studied with a LF spectral function.

From general properties within LF dynamics with a *fixed number* of degrees of freedom, a few relations are obtained among the six leading twist T-even TMDs, so that only three of the six T-even TMDs are independent. These novel relations are precisely predicted within LF dynamics, and could be experimentally checked to test the LF description of SIDIS, at least in the valence region.

#### REFERENCES

- [1] V. Barone, A. Drago, P. G. Ratcliffe, Physics Reports 359, 1 (2002)
- [2] S. Scopetta, Physical Review D 75, 054005 (2007)
- [3] A. Airapetian et al. The HERMES Collaboration, Physical Review Letters 94, 012002 (2005)
- [4] V. Yu. Alexakhin et al. COMPASS Collaboration, Physical Review Letters 94, 202002 (2005)
- [5] see talks at the "Transversity 2011" Lussino workshop, Il Nuovo Cimento C 35 (2012)
- [6] G. Cates et al., E12-09-018
- [7] A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Il Nuovo Cimento C 35, 101 (2012)
- [8] C. Ciofi degli Atti, E. Pace, G. Salmè, S. Scopetta, Physical Review C 48, R968 (1993)
- [9] R.B. Wiringa, V.G.J. Stocks, and R. Schiavilla, Phys. Rev. C 51, 38 (1995)
- [10] E. Pace, G. Salmè, S. Scopetta, and A. Kievsky, Phys. Rev. C 64, 055203 (2001)
- [11] C. Ciofi degli Atti et al., Physical Review C 46, R 159 (1992); A. Kievsky et al., Physical Review C 56, 64 (1997)
- [12] A. Kievsky et al., Nuclear Physics A 577, 511 (1994)
- [13] E. Pace, G. Salmè, S. Scopetta, Few-Body Systems, in press; E. Pace, G. Salmè, S. Scopetta, to be published
- [14] X. Qian, Physical Review Letters 107, 072003 (2011)
- [15] C. Ciofi degli Atti, L. Kaptari, Physical Review C 83, 044602 (2011)